Network Diagram
Critical Path Method
Programme Evaluation and Review Technique
and Reducing Project Duration

Prof. M. Rammohan Rao
Former Dean
Professor Emeritus
Executive Director, Centre for
Analytical Finance
Overview Of Presentation

- Work Breakdown Structure
- Project Network Diagram
  - Basic Rules and Convention
  - Critical Path Method
    - Forward Pass
    - Backward Pass
    - Slack
- Gantt Chart
- Programme Evaluation and Review Technique
- Reducing Project Duration
Work Breakdown Structure (WBS)

- Project work successively subdivided into smaller and smaller work elements
  - Leads to WBS

- WBS
  - Hierarchical outline (map) of the project
  - Ensures all work elements are identified
  - Final deliverables are divided into sub-deliverables
Work packages

- Lowest level of WBS
- Output oriented
- Defines work, i.e. what is to be done
- Identifies duration to complete a work package
- Identifies resources needed and quantity of resources
  - Time phased resource requirements
- Helps in identifying a person responsible for each work package.
Sometimes referred to as Gozinto Chart

Project Network

- Graphically shows the sequence, interdependencies
- Used to calculate / identify the
  - Start and finish time of each task
  - Completion time of the project
  - Critical path
    - Activities that should not be delayed if the project is to be completed by the calculated project completion time
Provides a basis for scheduling resources

Provides a basis for budgeting

Helps in monitoring the project as it progresses

Initially we will look at only duration of each task

Later we will look at reducing project duration by incurring additional costs to reduce activity times
<table>
<thead>
<tr>
<th>Activity</th>
<th>Description</th>
<th>Preceding Activity</th>
<th>Duration</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Design specifications and documentation</td>
<td></td>
<td>-</td>
</tr>
<tr>
<td>B</td>
<td>Production of prototype 1</td>
<td>A</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>Develop preliminary software</td>
<td>A</td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>Production of final prototype</td>
<td>B and C</td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>Final Version of software</td>
<td>D</td>
<td></td>
</tr>
<tr>
<td>K</td>
<td>Testing</td>
<td>F</td>
<td></td>
</tr>
</tbody>
</table>
From Work Package to Network (cont’d)

Constructing a Project Network

- **Activity**: an element of the project that requires time.
- **Event**: a point in time when an activity is started or completed. It does not consume time.

**Two Approaches**
- **Activity-on-Node (AON)**
  - Uses a node to depict an activity
- **Activity-on-Arrow (AOA)**
  - Uses an arrow to depict an activity
Constructing a Project Network (cont’d)

- **Terminology**
  - **Merge activity**: an activity that has two or more preceding activities on which it depends.
  - **Burst activity**: an activity that has more than one activity immediately following it (more than one dependent arrow flowing from it).
Constructing a Project Network (cont’d)

- Terminology

  - **Path**: a sequence of connected, dependent activities.

  - **Critical path**: the longest path through the activity network that allows for the completion of all project-related activities; the shortest expected time in which the entire project can be completed. Delays on the critical path will delay completion of the entire project.
Basic Rules to Follow in Developing Project Networks

- Networks typically flow from left to right.
- An activity cannot begin until all of its immediate predecessor activities are complete.
- Arrows indicate precedence and flow and can cross over each other.
- Identify each activity with a unique number; this number must be greater than its predecessors.
- Looping is not allowed.
- Conditional statements are not allowed.
- Use common start and stop nodes.
Illogical Loop

Network Information

<table>
<thead>
<tr>
<th>Activity</th>
<th>Description</th>
<th>Preceding Activity</th>
<th>Activity Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Application approval</td>
<td>None</td>
<td>5</td>
</tr>
<tr>
<td>B</td>
<td>Construction plans</td>
<td>A</td>
<td>15</td>
</tr>
<tr>
<td>C</td>
<td>Traffic study</td>
<td>A</td>
<td>10</td>
</tr>
<tr>
<td>D</td>
<td>Service availability check</td>
<td>A</td>
<td>5</td>
</tr>
<tr>
<td>E</td>
<td>Staff report</td>
<td>B, C</td>
<td>15</td>
</tr>
<tr>
<td>F</td>
<td>Commission approval</td>
<td>B, C, D</td>
<td>10</td>
</tr>
<tr>
<td>G</td>
<td>Wait for construction</td>
<td>F</td>
<td>170</td>
</tr>
<tr>
<td>H</td>
<td>Occupancy</td>
<td>E, G</td>
<td>35</td>
</tr>
</tbody>
</table>

Activity-on-Node Network

Network Computation Process

- **Forward Pass—Earliest Times**
  - How soon can the activity start? (early start—ES)
  - How soon can the activity finish? (early finish—EF)
  - How soon can the project finish? (expected time—ET)

- **Backward Pass—Latest Times**
  - How late can the activity start? (late start—LS)
  - How late can the activity finish? (late finish—LF)
  - Which activities represent the critical path?
  - How long can it be delayed? (slack or float—SL)
Forward Pass Computation

- Add activity times along each path in the network (ES + Duration = EF).
- Carry the early finish (EF) to the next activity where it becomes its early start (ES) *unless*...
- The next succeeding activity is a merge activity, in which case the largest EF of all preceding activities is selected.
Activity-on-Node Network Forward Pass

Backward Pass Computation

- Subtract activity times along each path in the network (LF - Duration = LS).
- Carry the late start (LS) to the next preceding activity where it becomes its late finish (LF) unless...
- The next preceding activity is a burst activity, in which case the smallest LS of all its immediate successor activities is selected as its LF.
Activity-on-Node Network Backward Pass

Determining Slack (or Float)

- Free Slack (or Float)
  - The maximum amount of time an activity can be delayed from its early start (ES) without affecting the early start (ES) of any activity immediately following it

- Total Slack
  - The maximum amount of time an activity can be delayed without delaying the entire project

- The critical path is the network path(s) that has (have) the least slack in common.
Activity-on-Node Network with Slack

Sensitivity of a Network

- The likelihood the original critical path(s) will change once the project is initiated.
  - Function of:
    - The number of critical paths
    - The amount of slack across near critical activities
Practical Considerations

- Network logic errors should be corrected
- Activity numbering
  - Usual convention: Predecessors have a lower number
- Use of computers to develop networks
- Calendar dates should be incorporated
  - Software allows this flexibility
- Multiple starts and multiple finishes
  - May be combined to have a single start and a single finish
- Multiple projects
  - Program or portfolio of projects
  - Software is available
Gantt Chart

- Developed by Henry L. Gantt in 1917
- PMBOK Guide: Bar Chart
- Planned start and finish dates for tasks displayed as bars against horizontal time scale
- Easy to read
- Can be used to show current status
- Useful for monitoring and making changes
- For large complex projects, difficult to follow multiple activity paths
Air Control Project

Air Control Project (cont’d)

Activity-on-Arrow Network Building Blocks

### Koll Business Center

#### County Engineers Design Department

<table>
<thead>
<tr>
<th>Activity</th>
<th>Description</th>
<th>Preceding Activity</th>
<th>Activity Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Application approval</td>
<td>None</td>
<td>5</td>
</tr>
<tr>
<td>B</td>
<td>Construction plans</td>
<td>A</td>
<td>15</td>
</tr>
<tr>
<td>C</td>
<td>Traffic study</td>
<td>A</td>
<td>10</td>
</tr>
<tr>
<td>D</td>
<td>Service availability check</td>
<td>A</td>
<td>5</td>
</tr>
<tr>
<td>E</td>
<td>Staff report</td>
<td>B, C</td>
<td>15</td>
</tr>
<tr>
<td>F</td>
<td>Commission approval</td>
<td>B, C, D</td>
<td>10</td>
</tr>
<tr>
<td>G</td>
<td>Wait for construction</td>
<td>F</td>
<td>170</td>
</tr>
<tr>
<td>H</td>
<td>Occupancy</td>
<td>E, G</td>
<td>35</td>
</tr>
</tbody>
</table>

Activity-on-Arrow Network

Program Evaluation Review Technique (PERT)

- Assumes each activity duration has a range that follows a beta distribution.
- PERT uses three time estimates for each activity: optimistic, pessimistic, and most likely activity durations.
- The average activity time is computed by the following formula:

\[ t_e = \frac{(a + 4m + b)}{6} \]

Where

\[ t_e = \text{average activity time based on the beta distribution} \]

\[ a = \text{optimistic activity time (1 chance in 100 of completing the activity earlier under normal conditions)} \]

\[ b = \text{pessimistic activity time (1 chance in 100 of completing the activity later under normal conditions)} \]

\[ m = \text{most likely activity time} \]
### PERT

**Table 8-1. Project Activity Times and Precedences**

<table>
<thead>
<tr>
<th>Activity</th>
<th>Optimistic Time</th>
<th>Most Likely Time</th>
<th>Pessimistic Time</th>
<th>Immediate Predecessor Activities</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>10</td>
<td>22</td>
<td>22</td>
<td>—</td>
</tr>
<tr>
<td>b</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>—</td>
</tr>
<tr>
<td>c</td>
<td>4</td>
<td>10</td>
<td>16</td>
<td>—</td>
</tr>
<tr>
<td>d</td>
<td>2</td>
<td>14</td>
<td>32</td>
<td>a</td>
</tr>
<tr>
<td>e</td>
<td>8</td>
<td>8</td>
<td>20</td>
<td>b, c</td>
</tr>
<tr>
<td>f</td>
<td>8</td>
<td>14</td>
<td>20</td>
<td>b, c</td>
</tr>
<tr>
<td>g</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>b, c</td>
</tr>
<tr>
<td>h</td>
<td>2</td>
<td>12</td>
<td>16</td>
<td>c</td>
</tr>
<tr>
<td>i</td>
<td>6</td>
<td>16</td>
<td>38</td>
<td>g, h</td>
</tr>
<tr>
<td>j</td>
<td>2</td>
<td>8</td>
<td>14</td>
<td>d, e</td>
</tr>
</tbody>
</table>

**Example:** Table 8.1 from Project Management: A Managerial Approach, Seventh Edition by Meredith and Mantel
Figure 8.13 from Project Management: A Managerial Approach, Seventh Edition by Meredith and Mantel.
<table>
<thead>
<tr>
<th>Activity</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
<th>e</th>
<th>f</th>
<th>g</th>
<th>h</th>
<th>i</th>
<th>j</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expected Time</td>
<td>20</td>
<td>20</td>
<td>10</td>
<td>15</td>
<td>15</td>
<td>14</td>
<td>4</td>
<td>11</td>
<td>18</td>
<td>8</td>
</tr>
</tbody>
</table>

Using expected times for activity durations:
Critical Paths: a---- d---- j
b---- e---- j

Project Completion Time : 43
Critical Activities : a, b, d, e and j
- Project completion time distribution depends upon the distribution of the duration (time) of each activity.
- Knowing the distribution of the time for each activity, we can find the distribution of project completion time.
  - Simulation
Activity time distribution

Project Completion time distribution
Reducing Project Duration

- Time Is Money: Cost-Time Tradeoffs
  - Reducing the time of a critical activity usually incurs additional direct costs.
    - Cost-time solutions focus on reducing (crashing) activities on the critical path to shorten overall duration of the project.
  - Reasons for imposed project duration dates:
    - Customer requirements and contract commitments
    - Time-to-market pressures
    - Incentive contracts (bonuses for early completion)
    - Unforeseen delays
    - Overhead and goodwill costs
    - Pressure to move resources to other projects
Alternatives

- Adding Resources
- Outsourcing Project Work
- Scheduling Overtime
- Establishing a Core Project Team
- Reducing Activity Times
- Reducing Project Scope
- Changing the project network
- Critical-Chain
Project Costs

- **Indirect Costs**
  - Costs that cannot be associated with any particular work package or project activity
    - Supervision, administration, consultants, and interest
  - Costs that vary (increase) with time
    - Reducing project time directly reduces indirect costs.

- **Direct Costs**
  - Normal costs that can be assigned directly to a specific work package or project activity
    - Labor, materials, equipment, and subcontractors
  - Reducing activity duration increases direct costs.
Project Cost—Duration Graph

- Find minimum total direct costs for selected project durations.
- Find total indirect costs for selected project durations.
- Sum direct and indirect costs for these selected project durations.
- Determining activities to shorten
  - Shorten the activities with the smallest increase in cost per unit of time.
Assumptions:

- The cost relationship is linear.
- Normal time assumes low-cost, efficient methods to complete the activity.
- Crash time represents a limit: the greatest time reduction possible under realistic conditions.
- Slope represents a constant cost per unit of time.
- All accelerations must occur within the normal and crash times.
### Cost—Duration Trade-off Example

<table>
<thead>
<tr>
<th>Activity ID</th>
<th>Slope</th>
<th>Maximum Crash Time</th>
<th>Direct Costs</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$20</td>
<td>1</td>
<td>Normal</td>
</tr>
<tr>
<td>B</td>
<td>40</td>
<td>2</td>
<td>Time</td>
</tr>
<tr>
<td>C</td>
<td>30</td>
<td>1</td>
<td>Normal</td>
</tr>
<tr>
<td>D</td>
<td>25</td>
<td>4</td>
<td>Crash</td>
</tr>
<tr>
<td>E</td>
<td>30</td>
<td>2</td>
<td>Normal</td>
</tr>
<tr>
<td>F</td>
<td>30</td>
<td>1</td>
<td>Crash</td>
</tr>
<tr>
<td>G</td>
<td>0</td>
<td>0</td>
<td>Normal</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Activity</th>
<th>Slope</th>
<th>Maximum Crash Time</th>
<th>Normal Time</th>
<th>Normal Cost</th>
<th>Crash Time</th>
<th>Crash Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$20</td>
<td>1</td>
<td>3</td>
<td>$50</td>
<td>2</td>
<td>$70</td>
</tr>
<tr>
<td>B</td>
<td>40</td>
<td>2</td>
<td>6</td>
<td>80</td>
<td>4</td>
<td>160</td>
</tr>
<tr>
<td>C</td>
<td>30</td>
<td>1</td>
<td>10</td>
<td>60</td>
<td>9</td>
<td>90</td>
</tr>
<tr>
<td>D</td>
<td>25</td>
<td>4</td>
<td>11</td>
<td>50</td>
<td>7</td>
<td>150</td>
</tr>
<tr>
<td>E</td>
<td>30</td>
<td>2</td>
<td>8</td>
<td>100</td>
<td>6</td>
<td>160</td>
</tr>
<tr>
<td>F</td>
<td>30</td>
<td>1</td>
<td>5</td>
<td>40</td>
<td>4</td>
<td>70</td>
</tr>
<tr>
<td>G</td>
<td>0</td>
<td>0</td>
<td>6</td>
<td>70</td>
<td>6</td>
<td>70</td>
</tr>
</tbody>
</table>

Cost-Duration Trade-off Example (cont’d)

Cost-Duration Trade-off Example (cont’d)

Cost—Duration Trade-off Example (cont’d)

![Network diagram showing project activities and costs]

Cost—Duration Trade-off Example (cont’d)

Summary Costs by Duration

<table>
<thead>
<tr>
<th>Project duration</th>
<th>Direct costs</th>
<th>+</th>
<th>Indirect costs</th>
<th>=</th>
<th>Total costs</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>450</td>
<td></td>
<td>400</td>
<td></td>
<td>$850</td>
</tr>
<tr>
<td>24</td>
<td>470</td>
<td></td>
<td>350</td>
<td></td>
<td>820</td>
</tr>
<tr>
<td>23</td>
<td>495</td>
<td></td>
<td>300</td>
<td></td>
<td>795</td>
</tr>
<tr>
<td>22</td>
<td><strong>525</strong></td>
<td></td>
<td><strong>250</strong></td>
<td></td>
<td><strong>775</strong></td>
</tr>
<tr>
<td>21</td>
<td>610</td>
<td></td>
<td>200</td>
<td></td>
<td>810</td>
</tr>
</tbody>
</table>

Thank You